

Reviewing Linear Systems — Final Answers

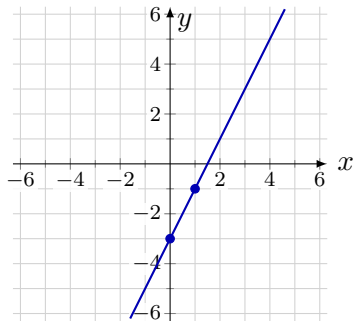
Final answers only.

Graph lines

1. The points used to draw each line are marked.

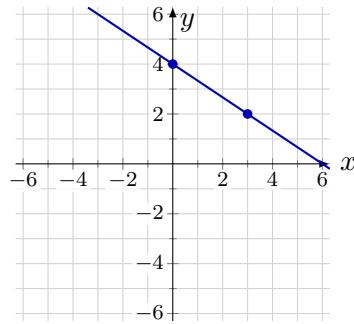
a) $y = 2x - 3$

slope 2, y -intercept -3



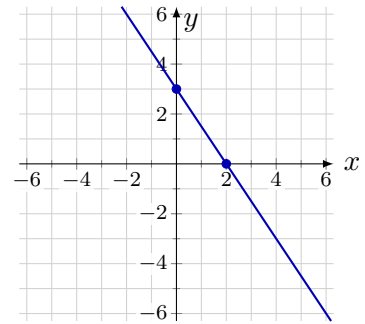
b) $y = -\frac{2}{3}x + 4$

slope $-\frac{2}{3}$, y -intercept 4



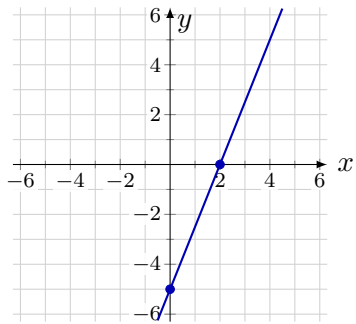
c) $3x + 2y = 6$

x -intercept 2, y -intercept 3



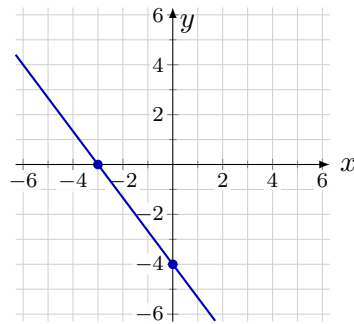
d) $5x - 2y = 10$

x -intercept 2, y -intercept -5



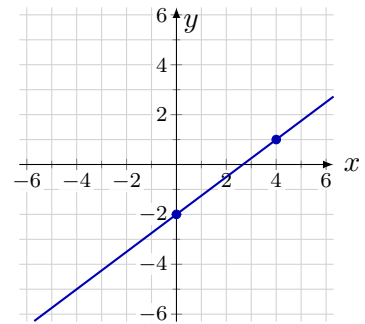
e) $4x + 3y + 12 = 0$

x -intercept -3 , y -intercept -4



f) $3x - 4y = 8$

slope $\frac{3}{4}$, y -intercept -2



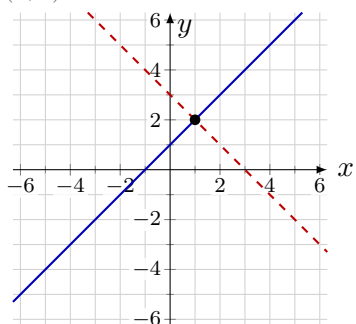
Solve linear systems by graphing

2. For e) and f), the lines cross between grid points. Accept any close estimate.

a) $y = x + 1$

$y = -x + 3$

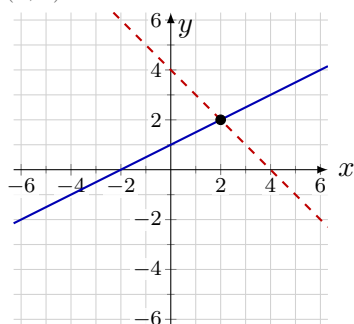
$(1, 2)$



b) $y = \frac{1}{2}x + 1$

$y = -x + 4$

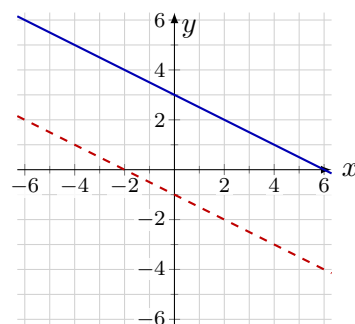
$(2, 2)$



c) $y = -\frac{1}{2}x + 3$

$x + 2y = -2$

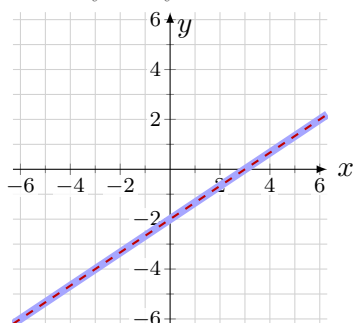
no solution



d) $y = \frac{2}{3}x - 2$

$2x - 3y = 6$

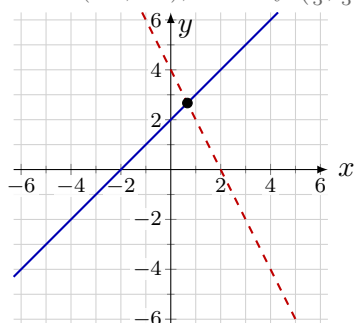
infinitely many solutions



e) $y = x + 2$

$y = -2x + 4$

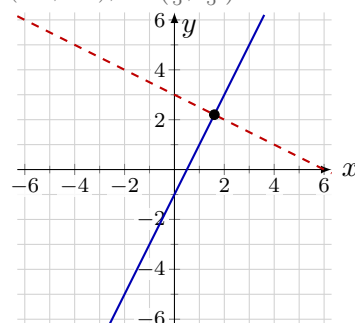
about $(0.7, 2.7)$; exactly $(\frac{2}{3}, \frac{8}{3})$



f) $2x - y = 1$

$x + 2y = 6$

$(1.6, 2.2)$, or $(\frac{8}{5}, \frac{11}{5})$



Solve linear systems algebraically

3. The method named is the more efficient one. Systems e) and g) are the special cases.

a) $(3, -3)$

substitution

b) $(2, 5)$

substitution

c) $(2, 1)$

elimination

d) $(3, 2)$

either method

e) no solution

$0x + 0y = 7$

f) $(2, -1)$

elimination

g) infinitely many

$0x = 0$

h) $(\frac{1}{2}, 1)$

elimination

Model problems with linear systems

4. Other choices of variables and units are possible. Solutions are given for reference; students were not asked to solve.

- a) Let x be the number of tacos sold. Let y be the number of burritos sold.

$$x + y = 85 \quad (\text{items sold}) \qquad 4x + 9y = 515 \quad (\text{sales, in dollars})$$

Solution: 50 tacos, 35 burritos

- b) Let x be the membership fee, in dollars. Let y be the fee for one visit, in dollars.

$$x + 8y = 106 \quad (\text{what Aiko paid}) \qquad x + 15y = 155 \quad (\text{what Ben paid})$$

Solution: membership \$50, each visit \$7

- c) Let x be the amount invested at 3%, in dollars. Let y be the amount invested at 5%, in dollars.

$$x + y = 8000 \quad (\text{money invested}) \qquad 0.03x + 0.05y = 340 \quad (\text{interest earned})$$

Solution: \$3000 at 3%, \$5000 at 5%

- d) Let x be the paddling speed in still water, in km/h. Let y be the speed of the current, in km/h.

$$x + y = 9 \quad (\text{speed with the current}) \qquad x - y = 3 \quad (\text{speed against the current})$$

Solution: paddling 6 km/h, current 3 km/h

- e) Let x be the length of the longer piece, in centimetres. Let y be the length of the shorter piece, in centimetres.

$$x + y = 300 \quad (\text{whole board}) \qquad x = y + 40 \quad (\text{lengths compared})$$

Solution: 170 cm and 130 cm

Or, with x and y in metres: $x + y = 3$ and $x = y + 0.4$ (1.7 m and 1.3 m). Mixing the units, as in $x + y = 3$ with $x = y + 40$, is the error to watch for.

- f) Let x be the number of dimes. Let y be the number of quarters.

$$x + y = 40 \quad (\text{number of coins}) \qquad 10x + 25y = 625 \quad (\text{value of the coins, in cents})$$

Solution: 25 dimes, 15 quarters

Or, with the value in dollars: $0.10x + 0.25y = 6.25$.

Check a solution

- 5.** The value of LS is given for each check, with RS in brackets where they differ.

- a) **(A)**: LS = 5 = RS. **(B)**: LS = -1 (RS = 13). b) **(A)**: LS = 12 = RS. **(B)**: LS = -1 = RS.
On line **(A)** only: not a solution. On both lines: the solution.

- c) **(A):** LS = 21 (RS = 4). **(B):** LS = 11 = RS.
On line **(B)** only: not a solution.
- d) **(A):** LS = 5 (RS = 6). **(B):** LS = 2 (RS = 3).
On neither line: not a solution.

- e) **(A)**: $LS = 3 = RS$. **(B)**: $LS = 9 = RS$. f) **(A)**: $LS = 1 = RS$. **(B)**: $LS = -3 = RS$.
On both lines, but **(B)** is $3 \times$ **(A)**, so the lines are coincident: one of infinitely many solutions. On both lines: the solution.

Mixed practice

For set-up questions, other variables and units are possible. Students were not asked to solve.

6. $(1, -2)$. Substitution: the first equation is already solved for y .
7. Let x be the number of jackets sold at the regular price. Let y be the number sold at the sale price.
 $x + y = 60$ (jackets sold) $80x + 50y = 4050$ (sales, in dollars)
Solution: 35 at the regular price, 25 at the sale price
8. No. If both variables are eliminated and the statement that is left is false, the lines are parallel and the system has no solutions. Jo should recheck the algebra, but $0 = 5$ can be a correct result.
9. $m = 4$ and $n = -3$ (substitute $x = 2$, $y = -1$: $2m - 3 = 5$ and $8 - n = 11$)
10. $(3, -2)$. Elimination: both equations are in the form $Ax + By = C$, and no variable has a coefficient of 1.
11. Let x be the speed of the plane in still air, in km/h. Let y be the speed of the wind, in km/h.
 $3(x + y) = 1800$ (distance with the wind) $3.6(x - y) = 1800$ (distance against the wind)
Solution: plane 550 km/h, wind 50 km/h
Or, dividing each distance by its time: $x + y = 600$ and $x - y = 500$.
12. Dan did not subtract the whole expression: $2x - (3x - 2) = 2x - 3x + 2$. Correct work: $-x + 2 = 5$, so $x = -3$ and $y = 3(-3) - 2 = -11$. The solution is $(-3, -11)$.
Check: $3(-3) - 2 = -11$ and $2(-3) - (-11) = 5$, so LS = RS in both equations.
13. No. A graph gives the exact solution only when the lines cross at a point that can be read exactly, such as a grid point. For example, $y = x + 2$ and $y = -2x + 4$ cross at $(\frac{2}{3}, \frac{8}{3})$, which a graph can only estimate. Substitution or elimination gives the exact values.
14. $k = -4$, because $6x - 3y = 12$ rearranges to $y = 2x - 4$, so the lines are coincident. For any other k , the lines have the same slope but different y -intercepts: they are parallel, and there are no solutions.
15. Let x be the number of minutes of calls in a month. Let y be the cost for the month, in dollars.
 $y = 20 + 0.10x$ (Plan A) $y = 5 + 0.25x$ (Plan B)
Solution: 100 minutes, \$30
The solution is the number of minutes for which both plans cost the same, and that cost.
16. Substituting gives $6y + 10 - 6y = 10$, or $10 = 10$, which is always true. There are infinitely many solutions: the lines are coincident.
17. Equations: its x - and y -values make both equations true (LS = RS in both). Graph: the point is on both lines, so it is where the lines cross.
18. Answers vary. a) same slope, different y -intercept, e.g. $y = -3x + 5$. b) an equivalent equation, e.g. $6x + 2y = 4$. c) any other line through $(1, -1)$, e.g. $y = x - 2$.
19. Let x be the length of the field, in metres. Let y be the width of the field, in metres.
 $2x + 2y = 1200$ (the fencing, in metres) $x = y + 30$ (length compared with width)
Solution: 315 m by 285 m
Or, with x and y in kilometres: $2x + 2y = 1.2$ and $x = y + 0.03$.